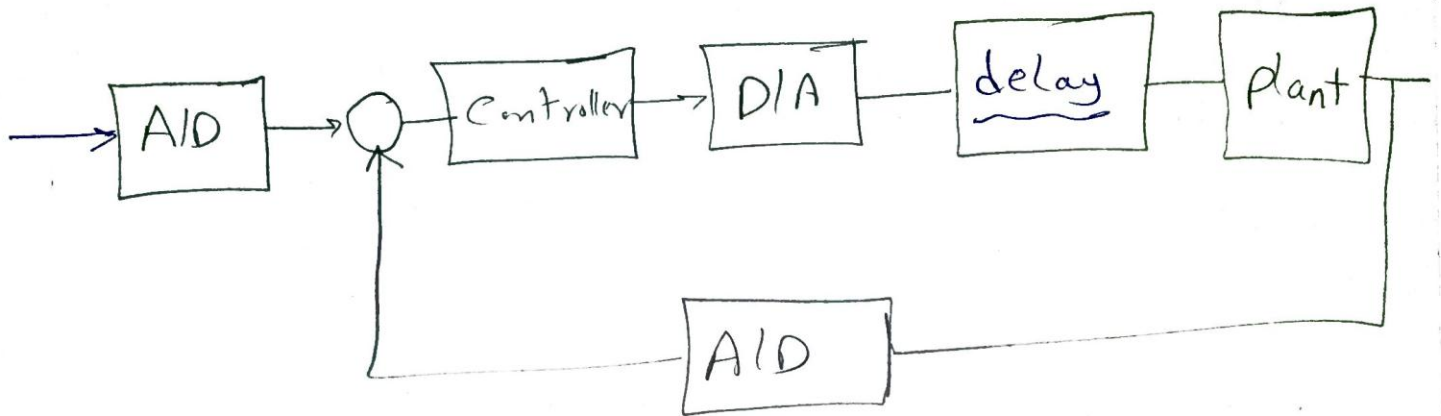
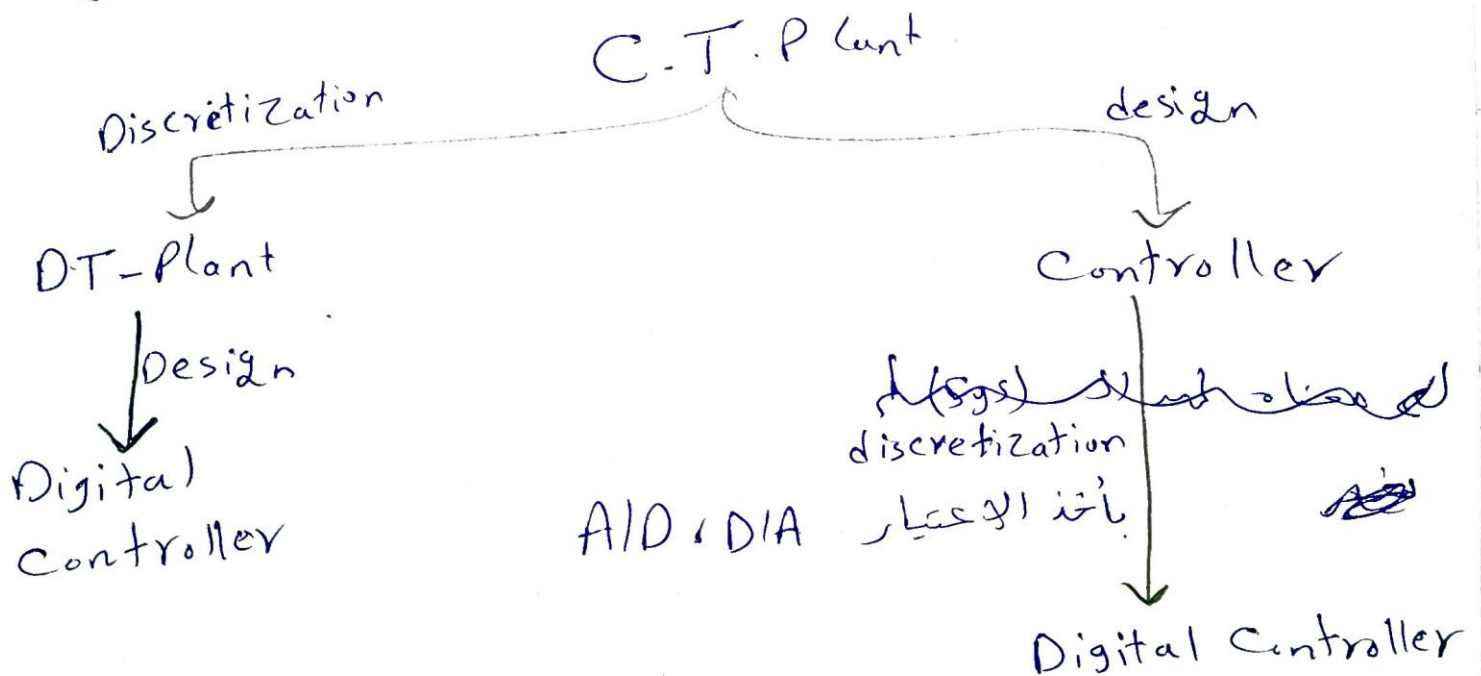


Digital Control sec 9



→ C.T. Plant
(continuous time plant)



For every control system

1) stability

2) Tracking (steady state error)

desired) \mathcal{U} (tracking) ~~low value~~ \rightarrow Sys. \mathcal{U}
old \rightarrow \mathcal{U}

3) Robustness (Disturbance Rejection)

\rightarrow steady state error

For unit step $e_{ss} = \frac{1}{1+K_p}$ For type 0

For unit ramp $e_{ss} = \frac{1}{K_v}$ For type 1

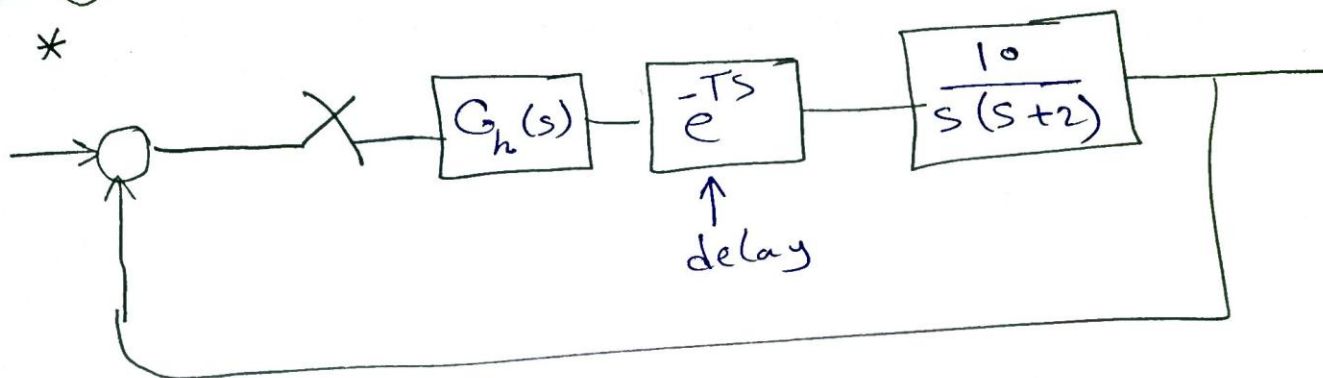
For ~~unit~~ parabolic $e_{ss} = \frac{1}{K_a}$ For type 2

as \rightarrow (s-domain) \rightarrow (Final value) $\mathcal{U} \leftarrow$
(z-domain) $\rightarrow \mathcal{U}$

$$K_p = \frac{1}{T} \lim_{z \rightarrow 1} \overline{GH(z)}$$

$$K_v = \frac{1}{T} \lim_{z \rightarrow 1} (1-z^{-1}) \overline{GH(z)}$$

$$K_a = \frac{1}{T^2} \lim_{z \rightarrow 1} (1 - \bar{z}^{-1})^2 \bar{G}H(z)$$



$$G_H(s) = \left(\frac{1 - e^{-Ts}}{s} \right) e^{-Ts} * \frac{10}{(s+2)s}$$

$$\bar{G}H(z) = \bar{z}^{-1} (1 - \bar{z}^{-1}) \underbrace{Z \left[\frac{10}{s^2(s+2)} \right]}$$

unit delay $\leadsto e^{-Ts}$; unit delay in z-Transform = \bar{z}

$$\bar{z} Z \left[\frac{A_1}{s^2} + \frac{B}{s} + \frac{C}{s+2} \right]$$

$$\bar{z} \left[\frac{5}{s^2} - \frac{2.5}{s} + \frac{2.5}{s+2} \right]$$

$$\bar{z} \left[5t - 2.5 + 2.5 e^{-2t} \right]$$

$$Z \left[5KT - 2.5 + 2.5 e^{-2KT} \right]$$

$$= \frac{5ZT}{Z-1} - 2.5 \frac{Z}{Z-1} + 2.5 \frac{Z}{Z - e^{-2T}}$$

$$\cancel{GH(z)}$$

$$\cancel{GH(z)}$$

$$GH(z) =$$

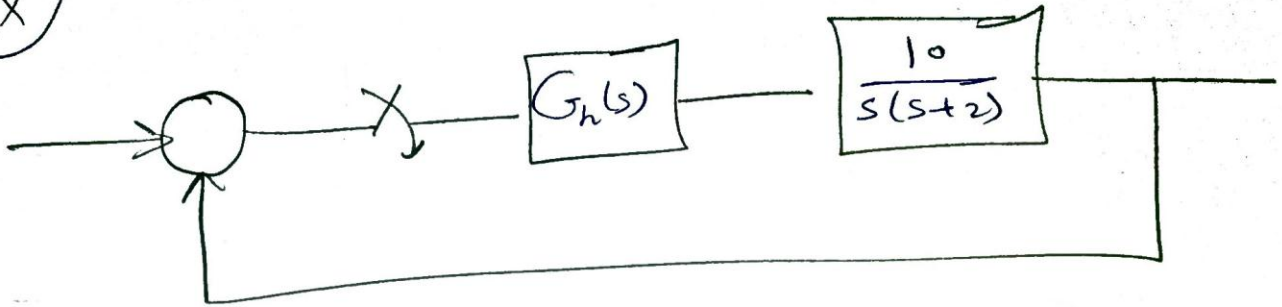
$$\frac{Z^{-1}(1-Z^{-1}) \left[5ZT(Z - e^{-2T}) - 2.5(Z-1)(Z - e^{-2T}) + 2.5(Z-1)^2 \right]}{(Z-1)^2 (Z - e^{-2T})}$$

$$GH(z) \Big|_{C.L} = \frac{G(z)}{1 + GH(z)}$$

(type 2) ∞ Sys. ال (physically) ←

لكن في ال " مفيد حاجة لسه أشيل (root) في البنية مع النظام لكن رياضياً عادي.

ex



stability

→ clc eqn.

Stability → routh (in cont.)
 → ~~mat~~ Jury test
 → Check system poles.

Cont: برده سکتا دایز (discrete) ← جزئی ۱
 r-domain } z-domain بیحول

$$\frac{\text{Routh}}{Z = e^{\frac{T}{s}}} = \frac{e^{\frac{T}{2}s}}{(-T/2)s} \cdot \frac{1 + \frac{T}{2}s}{1 - \frac{T}{2}s} \cdot \frac{1+r}{1-r}$$

map any $z \rightarrow \frac{1+r}{1-r}$

Jury test

$$1) F(1) > 0$$

cle eq $|_{z=1}$

$$2) (-1)^n F(-1) > 0$$

n : order

$$3) a_0 < a_n$$

→ Jury matrix

$$\begin{array}{ccccccc} & a_0 & a_1 & \dots & & & a_n \\ & & & & & & a_0 \\ a_n & a_{n-1} & \dots & & & & \\ \hline & | a_0 & a_n | & | a_0 & a_{n-1} | & \dots & \\ & | a_n & a_0 | & | a_n & a_1 | & \dots & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{array}$$

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ex)

$$P(z) = z^2 + 1.703z + 1.620 \quad \text{c/c}$$

$$1) P(1) = 1 + 1.703 + 1.620 = 7.0$$

$$2) (-1)^n P(-1) = (-1)^2 [1 - 1.703 + 1.620] = 7.0$$

$$3) a_0 = 1.620 \quad a_0 > a_2$$

$$a_{2n} = 1$$

→ system unstable

c/c eqn:

$$z^3 - 1.368z^2 + 0.368z + 0.368Kz + 0.264K = 0$$

$$F(1) = 0 \rightarrow 1 - 1.368 + 0.368 + 0.368K + 0.264K = 0$$

$$\boxed{n=3}$$

$$(-1)^n F(-1) = 0 \rightarrow (-1)^3 - 1.368 - 0.368 - \dots$$

$$\rightarrow a_0 < a_3$$

$\downarrow \quad \quad \downarrow$
 $0.264K \quad \quad 1$

$$\boxed{7}$$

المعروف في الفصل ٣ شروط السابقة كل شرط
 هيكل range J K فنحسب التقاطع ما بينهم

Jury matrix

$$\begin{array}{cccc}
 0.264K & (1+K)0.368 & -1.368 & 1 \\
 1 & -1.368 & (1+K)0.368 & 0.264K \\
 \hline
 (0.264K)^2 - 1 & - & -0.264 \times 1.368K - (1+K)0.368 & -
 \end{array}$$

← ك. د. أربع معادلات فنحسب التقاطع ما بينهم.

⇒ (range for stability check) ~~☆~~

→ you better use Routh array.

solve by Routh

$$Z^3 - 1.368 Z^2 + 0.368 Z + 0.368 K Z + 0.264 Z = 0$$

$$Z = \frac{1+r}{1-r}$$

$$\begin{aligned}
 & \frac{(1+r)^3}{(1-r)^3} - 1.368 \left(\frac{1+r}{1-r} \right)^2 + 0.368 \left(\frac{1+r}{1-r} \right) + \\
 & 0.368 K \left(\frac{1+r}{1-r} \right) + 0.264 \left(\frac{1+r}{1-r} \right)
 \end{aligned}$$

(8)

المعادلة المميزة \rightarrow حساب \rightarrow (Routh)

* 2nd order eqn. \rightarrow Jury

* Check stability \rightarrow Routh

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